

**FINITE ELEMENT SIMULATION FOR MICROWAVE DEVICES  
APPLICATIONS TO MICROWAVE D.R. FILTERS**

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**ABSTRACT**

Rigorous numerical analysis of an axis symmetrical  $TM_{0,1,\delta}$  dielectric resonators mode filter is presented.

Two dimensional (2D) and three dimensional (3D) finite element method (F.E.M.) is applied to compute the exact scattering matrix parameters (Taking into account the excitation probes) of this device. A sensitivity analysis is also performed and permits easier filter tuning. Experimental results are given and are in good agreement with theoretical ones.

**INTRODUCTION**

Dielectric resonators (DRs) are widely used in microwave applications, particularly in microwave filters, due to their desirable properties and their commercial availability at reasonable price.

The high electrical performances required for microwave filters achievement with DR, need rigorous analysis able to provide their behaviour with high degree of accuracy particularly for several parameters results of the filter synthesis like the coupling between DRs, the input and output coupling coefficients, the proper mode resonant frequency ( $f_0$ ) or the unloaded quality factor ( $Q_0$ ).

Up to now [1], [2], [3], [4], the study of each parameters is generally done separately.

In order to take into account all DR influences on each other and then to study the structure completeness, a solution is to use the 2 or 3 dimensional forced oscillations finite element method (F.E.M.).

In this paper, we only present results concerning axis symmetrical DRs structures which necessitate only the use of the two dimensional F.E.M. Section 1 describes succinctly this method used to solve electromagnetic forced or free oscillations problems, applied to DRs devices characterization. In section 2, an example of such an analysis is presented. It consists to evaluate the scattering parameters of a  $TM_{0,1,\delta}$  DR mode coupled with two coaxial probes. Theoretical and experimental results are given. Then, a sensitivity analysis will be performed to show the geometrical parameters influence on the device responses. The method may be extended to optimize all structures. The analysis is then extended in section 3 to a two DRs axis symmetrical structure excited by two coaxial probes.

**I - FINITE ELEMENT ANALYSIS**

The two dimensional finite element method previously introduced [5], is used to analyze axis symmetrical structures as shown in figure 1. The structure is composed of  $N$  homogeneous, linear, isotropic and loss-less sub mediums, bounded with a circular metallic enclosure. It is excited by two coaxial probes which may support both T.E.M. modes and the first higher order  $TM_0$  modes. Those excitations are represented by magnetic and electric surface currents imposed in the reference planes  $P_1$  and  $P_2$ .

In order to calculate the electromagnetic field and the scattering coefficients of this device, we solve the following wave equation deduced from Maxwell ones applied to distributions [5], [6], [7].

$$\iiint_V \left( \frac{1}{\epsilon_i} \{ \text{rot} \} \{ H \} \right) \{ \text{rot} \} \phi \cdot dV - k^2 \iiint_V H \cdot \phi \cdot dV \\ = -j \omega \epsilon_0 \sum_{k=1}^2 J_{mp_k} \cdot \phi dS_{p_k} \quad (1)$$

where :  $H$  is the magnetic field vector

$\epsilon_i$  is the relative dielectric constant of medium  $i$

$$k = \omega \sqrt{\epsilon_0 \mu_0}$$

$V$  is the volume of the structure

$S_{p_k}$  is the surface of port  $k$

$J_{mp_k}$  represent the magnetic current at port  $k$

$\phi$  is a test function

The second member of the equation (1) represents the excitation of the structure and it can be developed to introduce voltage currents waves  $a_k$  and  $b_k$  in the formulation. After a numerical processing, we obtain a linear system to solve.

Knowing the frequency at which the structure is acting, the computation of this system yields to :

- the scattering matrix parameters in references planes  $P_1$  and  $P_2$
- the magnetic field vector in each medium of the structure

In the particular case where the references planes are short circuited,  $J_{mp_k} = 0$  and the second member of (1) cancels. This free oscillation analysis yields to eigen modes resonant frequency and field distribution.

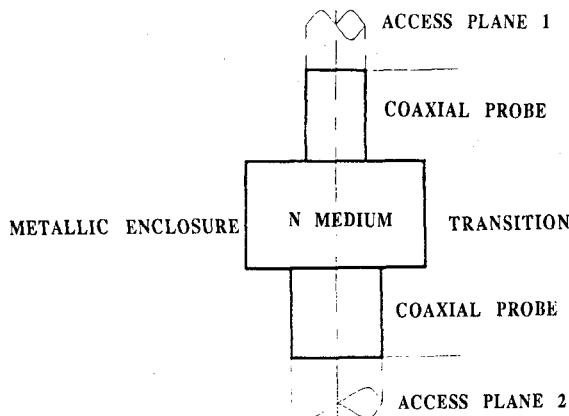


Figure 1 : Transition between coaxial probes

## II - ANALYSIS OF A STRUCTURE COMPOSED OF A D.R. COUPLED WITH TWO PROBES

### A) - Structure presentation

The first structure we are going to study is shown in figure 2. A DR of radius  $r=11$  mm, height  $L=7$  mm and permittivity  $\epsilon_r=36,8$  is enclosed in a perfectly conducting cylindric cavity of radius  $r=2r$  and height  $L=3L$ . This DR is supported by a concentric low relative dielectric constant material ring (e.g. Teflon).

The DR is excited in its electrical dipolar  $TM_{0,1,0}$  mode, excitation which can be achieved by mean of coaxial probes mounted axially in the center of the metallic cavity. The reference planes  $P_1$  and  $P_2$  are chosen far enough from the discontinuities, so that the higher TM modes of the probes vanished in those planes.

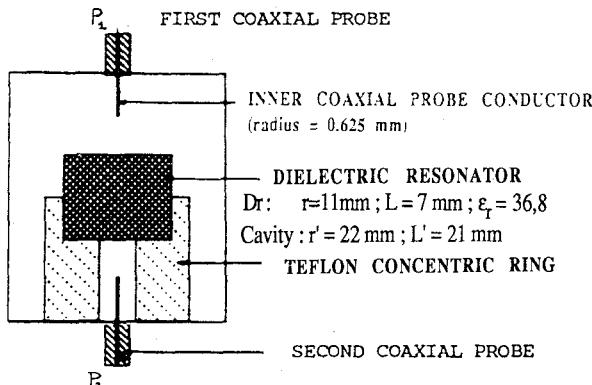


Figure 2 : A dielectric resonator coupled with two coaxial probes

### B) - Theoretical and experimental results

The F.E.M. described previously is used to compute the scattering matrix parameters in the reference planes  $P_1$  and  $P_2$  of the structure shown in figure 2.

The theoretical  $S_{21}$  modulus variation as a function of the frequency for a 6 mm depth penetration probes is presented in figure 3. Since the DR is supposed loss-less, the theoretical external quality factor  $Q_e$  can be obtained from the pass band response of the one pole DR filter (figure 3),

applying :

$$Q_e = \frac{\omega_0}{\Delta\omega} \quad (2)$$

where  $\omega_0$  is the resonant pulsation,  $\Delta\omega$  is the pass band width.

Then, to evaluate the loaded quality factor  $Q_L$ , we first evaluate the unloaded quality factor  $Q_0$  by solving the free oscillation equation and knowing  $Q_e$  from the theoretical response, we deduce  $Q_L$  from :

$$Q_L^{-1} = Q_0^{-1} + Q_e^{-1} \quad (5)$$

A computation of theoretical and experimental  $Q_L$  factor for different coaxial probe depth penetration values is presented in figure 4. We can see a good agreement between those theoretical results and experimental ones. Figure 5 shows the  $|S_{21}|$  parameter variations as a function of the frequency for varius probe depth penetrations.

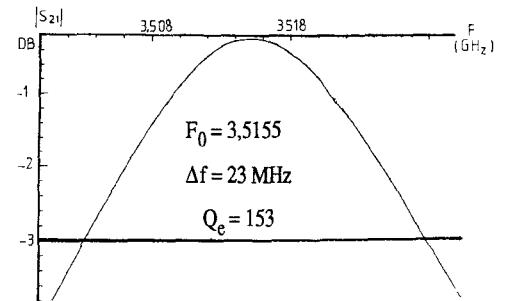


Figure 3 : Modulus of the transmission coefficient  $S_{21}$  as a function of the frequency for a 6 mm depth penetration probes

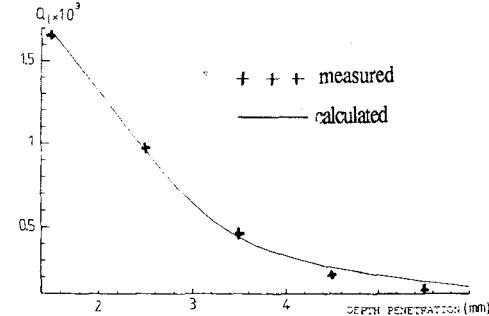


Figure 4 : Loaded quality factor

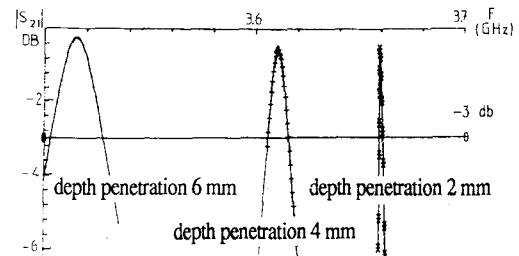


Figure 5 : Modulus of the transmission coefficient  $S_{21}$  as a function of the frequency for varius probe depth penetrations

### C) - Sensitivity analysis

Filter designers may study the influences of the structures dimensions on the filter responses to know the degree of accuracy required on them. The theoretical sensitivity calculation after the resolution of the linear system has already been presented in [5].

For the structure given figure 2, it yields to the scattering matrix terms derivate as a function of :

- geometrical parameters : cavity height and radius - axis shifting resonators - probe depth penetration
  - resonator height and radius.
- electrical parameters like : RD permittivity - support permittivity

Note that it is also possible to compute sensitivity relative to many parameters at the same time. The computer algorithm permits to obtain in a few seconds accurate results for structures slightly different from the original one.

We only present here a computation example which permit to appreciate the accuracy of the method. Figure 6 presents a interpolated curve for  $\Delta\epsilon_r = +0,1$  and the computed response for  $\epsilon_r = \epsilon_{r1} + \Delta\epsilon_r$ . We can note a very good agreement between the results obtained by using the interpolation software and those obtained by using a new mesh of the structure and running again the program.

### D) - Higher modes analysis

Up to now, we are working with the first  $TM_0$  DR mode. In this chapter, we will see that the same software can also be used to evaluate the electrical parameters of the second  $TM_0$  DR mode. The response obtained in this case is shown in figure 7. We can verify that the coupling factor is lower than the one obtained for the first  $TM_0$  mode for the same probes depth penetration, as the first  $TM_0$  mode energy is less concentrated in the DR than thus of the second one.

## III - ANALYSIS OF A TWO DIELECTRIC RESONATORS STRUCTURE

### A) - Structure presentation

Two dielectric resonators are included in a perfectly conducting cylindric shield. They are supported by a concentric Teflon ring. The excitation is achieved by coaxial probes mounted axially in the center of cavity (figure 8).

### B) - Theoretical results

Figure 9 presents some responses obtained for varius distances d between DRs as a function of the frequency. We can verify that the coupling coefficient [1] decreases when d increases. This coefficient solution of the forced oscillation problem can be compared to this obtained by solving the free oscillation equation and evaluate from (4).

$$k_0 = \frac{f_{0e}^2 - f_{0o}^2}{f_{0e}^2 + f_{0o}^2} \quad (6)$$

where  $f_{0e}$  and  $f_{0o}$  are respectively frequencies of even and odd modes of the DRs structure (figure 10).

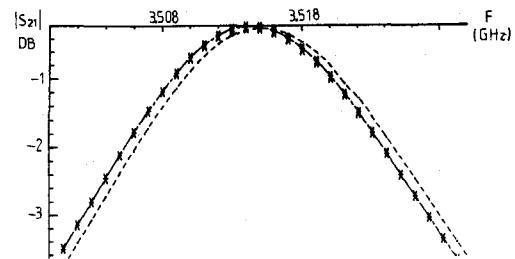


Figure 6 : Modulus of the transmission coefficient  $S_{21}$  as a function of the frequency

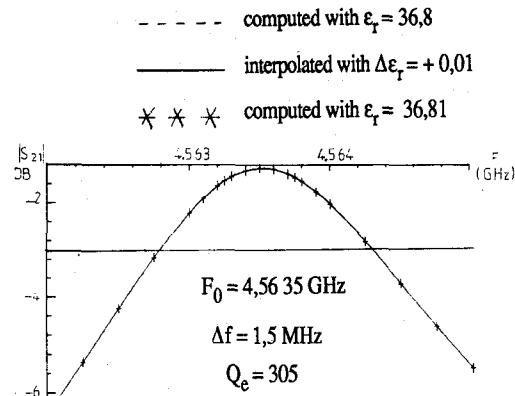


Figure 7 : Modulus of the transmission coefficient  $S_{21}$  as a function of the frequency of the second  $TM_0$  mode

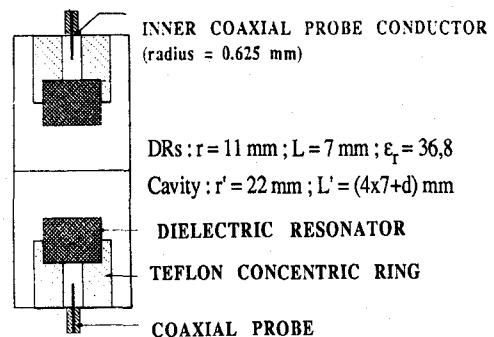


Figure 8 : Two dielectric resonators coupled with two coaxial probes

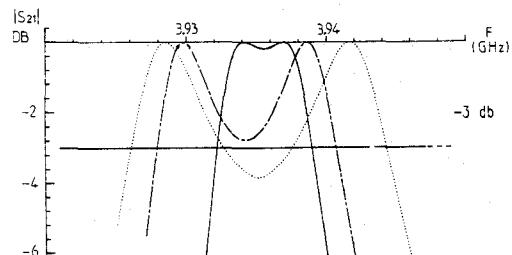


Figure 9 : Modulus of the transmission coefficient  $S_{21}$  as a function of the frequency for varius distances d between DR

- d = 80 mm
- d = 70 mm
- d = 64 mm

Finally, some results concerning the DR height and radius influences on the  $|S_{21}|$  responses are presented on figure 11. These results show that a complete analysis sensitivity is very important for filters adjustments and for the experimental response prediction from manufacturer performances.

#### IV - CONCLUSION

The forced oscillations two dimensionnal F.E.M. presented in this paper leads to the scattering matrix parameters of a cylindrical DR structure excited on the  $TM_{0,m,p}$  mode using axial coaxial probes.

The sensitivity analysis is used to observe the geometrical variation influence on those S parameters and can be applied to the structure optimisation. We have now to use the three dimensionnal F.E.M. to analyse asymmetric structures and, for example, the excitation of TE or HEM DR modes achieved by coaxial loops or microstrip lines.

#### B I B L I O G R A P H I E

[1] K.A. ZAKI, C. CHEN

"Coupling of non-axially symmetric hybrid modes in dielectric resonators"  
IEEE Trans. MTT-35, n°12, December 1987, pp.1136-1142

[2] A.P.S. KHANNA

"Dielectric resonator of  $HE_{1,1,\delta+1}$  mode coupling to a shielded microstrip line"  
IEEE MTT-S Digest, 1983, pp.527-529

[3] S. VIGNERON, P. GUILLOON

"Input impedance and loaded quality factor of a probe exciting dielectric resonators"  
IEE Proceedings, Vol.135, pt.H, n°3, June 1988

[4] D. KAJFEZ and P. GUILLOON

"Dielectric resonators"  
Artech House 1986

[5] E. MAROUBY, M. AUBOURG,  
P. GUILLOON

"Application of finite element method to the study of transition between coaxial lines"  
IEE Proceedings, Vol.137, pt.H, n°4, August 1990

[6] A. KONRAD

"Vector variational formulation of electromagnetic field in anisotropic media"  
IEEE Trans. on Microwave Theory, Vol. MTT-24, pp.553-559,  
September 1976

[7] R. FERRARI and G. MAILLE

"Three dimensional finite element method for solving electromagnetic problems"  
Electronic Letters, Vol.14, n°15, pp.467-468, July 1978

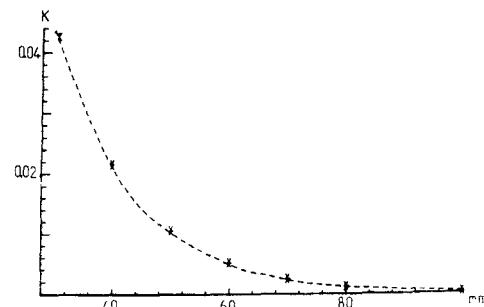


Figure 10 : The inter resonators coupling coefficient as a function of the distance  $d$  between DR  
free oscillation  
forced oscillation

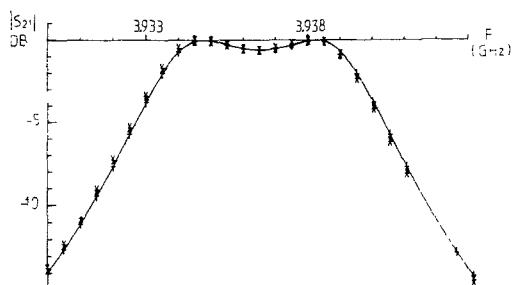


Figure 11 : Modulus of the transmission coefficient  $S_{21}$  as a function of frequency  
— computed with  $r = 11$  mm and  $L = 7$  mm  
\* \* \* interpolated with  $\Delta r = +0,01$   
+ + + interpolated with  $\Delta L = +0,001$